CRITERIAL EQUATIONS FOR CALCULATING THE FRICTION AND HEAT EXCHANGE AT A VERTICAL POROUS SURFACE WITH COMBINED CONVECTION

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Formulas are obtained for calculating the surface friction and heat exchange at a vertical porous surface with injection (suction) under combined convection conditions.

In the transfer of heat from a heated surface to a medium moving around it, forced and free convection are of predominant importance. Until recently, theoretical and experimental investigations on this theme were limited to the form of one of the transfer mechanisms. It is obvious that heat is transferred by the two mechanisms acting simultaneously. Analysis of the experimental data in [1] shows that although the motion in the experiments was determined principally by a forced current, when comparing their results with analytical calculations, and taking account only of forced convection, the powerful influence of free convection can be seen.

The effect of buoyancy on forced convection in the case of laminar flow around a vertical plate was analyzed by Sparrow and Gregg [2]. They showed that the solution for the boundary layer can be represented in the form of a power series in Gr/Re^2, the first term of which is the solution for purely convective flow. In [3], the effect of buoyancy on forced convection is considered for laminar flow, and also the effect of forced convection on the purely free motion in the case of flow around a vertical surface. Solutions are obtained by expansion in series of the flow and temperature functions with respect to the parameter Gr/Re^2 in the first case, and with respect to the parameter $Re/Gr^{1/2}$ in the case of treatment of free convection. For small values of the defining parameters, profiles of velocities and temperatures, shear stresses and heat transfer are given. The effect of natural convection on the frictional stress for a gas with forced flow is investigated also by an approximate method [4].

The method used in this report for investigating the problem posed, concerning combined convection, consists in the conversion of a system of differential equations in partial derivatives for a boundary layer expressing the laws of conservation of mass, momentum, and energy, to ordinary differential equations by means of a similar conversion for specified laws of distribution of the surface temperature and flow velocity at the outer edge of the boundary layer.

The combined convection near a vertical surface is described by the system of differential equations of a laminar boundary layer. For directional coincidence of the forced and free convection and the constant properties of the medium, with the exception of density, depending on the temperature in the expression for the buoyancy and without taking viscous dissipation into account, the equations have the following form:

$$\begin{gathered}\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,\\ u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{dp}{dx}+g\beta(T-T_\infty),\\ u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=a\frac{\partial^2 T}{\partial y^2}\end{gathered}\tag{1}$$

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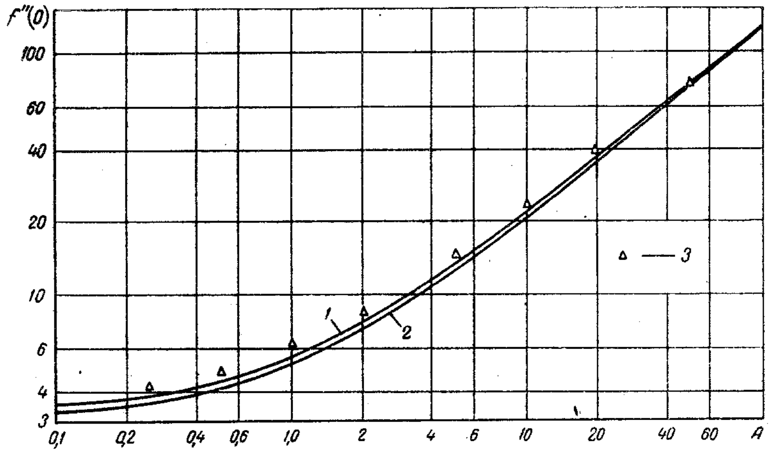

Fig. 1. Surface friction under combined convection conditions (Pr = 0.7).

with the boundary conditions

$$y = 0:\ u = 0,\ v = v_w,\ T = T_w,$$
$$y = \infty:\ u = U_\infty,\ T = T_\infty. \tag{2}$$

If we consider convective flow and heat transfer in the boundary layer, under conditions which specify the velocity distribution in the flow remote from the surface and the temperature of the surface

$$U_\infty = cx^m,$$
$$T_w = T_\infty + Bx^n, \tag{3}$$

where C, B, m, and n are constant quantities.

The problem (1)-(3) reduces to a system of two ordinary equations relative to $f(\eta)$ and $\theta(\eta)$ by the introduction of the flow function

$$\Psi = c_2 x^\alpha f(\eta) \tag{4}$$

and the independent variable

$$\eta = c_1 y x^\beta, \tag{5}$$

where the quantities α, β, c_1, and c_2 are determined in the following way

$$\alpha = \frac{m+1}{2} = \frac{n+3}{4},\quad \beta = \frac{m-1}{2} = \frac{n-1}{4},$$
$$c_1 = \frac{1}{2}\left(\frac{C}{\nu}\right)^{0,5},\quad c_2 = (C\nu)^{0,5}.$$

Conversion to ordinary differential equations is possible under conditions such that

$$n = 2m - 1. \tag{6}$$

The system of equations (1) in dimensionless form is written as

$$f'''(\eta) + (m+1) f(\eta) f''(\eta) - 2mf'^2(\eta) + 8\left[m + \frac{Gr}{\mathrm{Re}^2}\theta(\eta)\right] = 0,$$
$$\theta''(\eta) + \Pr\left[(m+1) f(\eta)\theta'(\eta) - (4m-2) f'(\eta)\theta(\eta)\right] = 0 \tag{7}$$

with the boundary conditions

$$\eta = 0:\ f'(0) = 0,\ f_w = \mathrm{const},\ \theta = 1,$$
$$\eta = \infty:\ f'(\infty) = 2,\ \theta = 0,$$

where $\theta(\eta) = (T-T_\infty)/(T_W-T_\infty)$ and the prime denotes differentiation with respect to η. This system contains the Prandtl number and $A = Gr/Re^2$ as parameters. The parameter A defines the effect of free convection on the surface friction and heat exchange under combined convection conditions.

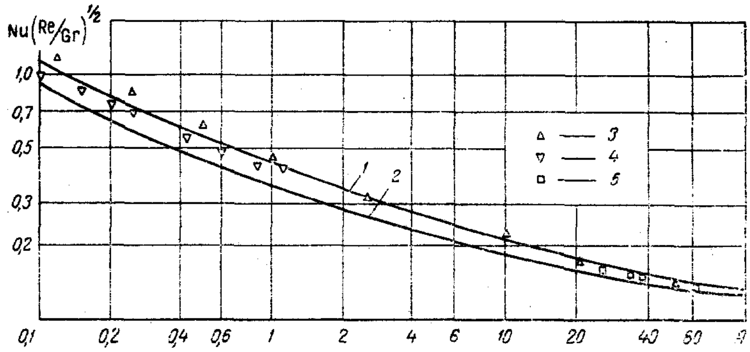

Fig. 2. Heat exchange with combined convection (Pr = 0.7).

The system of equations (7) is solved numerically on a computer for Pr = 0.7; m = 0.5 and n = 0, and for a wide range of the parameter A in the case of injection (suction) it is solved in [5]. The results of the calculations are used for obtaining the criterial equations.

The surface friction on a vertical porous surface is determined by the expression [6]

$$\tau_w = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0} - \rho v_w U_\infty,$$

which, in transformed variables, permits the dimensionless coefficient of friction to be obtained

$$c_f \mathrm{Re}^{1/2} = \frac{1}{2} f''(0) + (m+1) f_w, \tag{8}$$

where $c_f = 2\tau_w/(\rho U_\infty^2)$, and $f_w = -(2v_w/(m+1)U_\infty)\sqrt{\mathrm{Re}}$ is the injection (suction) parameter. Based on processing of the results of the numerical calculation, the formula

$$f''(0) = (3.578 + 2.654A^{0.835})(1 + 0.25 f_w A^{0.38}), \tag{9}$$

is obtained, by means of which the surface friction in the case of combined convection and injection (suction) is determined. Figure 1 shows the results of the calculation by formula (9) for an impermeable surface (curve 1) and injection (curve 2 for $f_w = -0.2$). Here also, the results of the numerical calculation in [7] for an impermeable surface are plotted by the points 3.

The thermal flux at the surface, in the case of combined convection, is determined by the law

$$q = -\lambda \left(\frac{\partial T}{\partial y}\right)_{y=0}.$$

Introducing the dimensionless Nusselt number,

$$\mathrm{Nu} = \frac{\alpha x}{\lambda}, \text{ where } \alpha = \frac{q}{T_w - T_\infty},$$

for the local efficiency, we obtain

$$\mathrm{Nu} = -\frac{1}{2}\theta'(0)\,\mathrm{Re}^{1/2}, \tag{10}$$

where $\theta'(0)$ is evaluated in [5] for different values of the defining parameters A and f_w.

In order to calculate the heat exchange in the case of combined convection, based on the results of the numerical integration of system (7), the criterial equation

$$\mathrm{Nu}\left(\frac{\mathrm{Gr}}{\mathrm{Re}^2}\right) = \frac{0.44 f_w + 0.47}{A^{0.12 f_w + 0.34}} \tag{11}$$

is obtained.

A comparison of the result of the calculation by formula (11) with the experimental and calculated data of other authors is given in Fig. 2, where curves 1 and 2 represent the heat exchange when $f_w = 0$ and -0.2 respectively; 3 are the calculated [7], 4 are the experimental [3] and 5 are the calculated data [8] for an impermeable surface.

It is interesting to note that in the paper by S. Eshgi [8], an approximate solution for mixed convection is obtained, defined by the parameter $Re/Gr^{1/2}$, when the forced flow affects the free flow. Thus, when investigating the heat exchange under conditions of combined convection, there are two approaches: study of the effect of free motion on the forced convection and vice versa — the effect of the forced flow on the natural convection. However, by plotting the results of heat exchange in the coordinates $Nu(Re/Gr)^{1/2}$ and Gr/Re^2, both approaches are shown to be identical, i.e., when investigating combined convection, the study can be restricted to only the effect of free convection on forced convection.

NOTATION

x, y	are the coordinates;
u, v	are the components of the velocity along the axes;
g	is the acceleration due to gravity;
T	is the temperature;
ν	is the kinematic viscosity;
β	is the coefficient of thermal expansion;
a	is the coefficient of thermal conductivity;
ρ	is the density;
p	is the pressure;
U_∞	is the external flow velocity;
η	is the independent variable;
τ	is the shear stress;
λ	is the coefficient of thermal conductivity;
A	is the parameter of the effect of natural convection on forced convection;
f_W	is the injection (suction) parameter;
θ	is the dimensionless temperature;
$Gr = g\beta(T_W - T_\infty)x^3/\nu^2$, $Re = Ux/\nu$, $Pr = \nu/a$, $Pr = \nu/a$, $Nu = \alpha x/\lambda$	are the Grasshof, Reynolds, Prandtl, and Nusselt numbers, respectively;

Subscripts

w denotes values at the surface;

∞ denotes values at a large distance from the surface.

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